Is the Pioneer Anomaly a Counter Example to the Dark Matter Hypothesis?

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Abstract The Hubble law is extended to massive particles based on the de Broglie wavelength. Due to the expansion of the universe the wavelength of an unbound particle would increase according to its cosmological redshift. Based on the navigation anomalies of the Pioneer 10 & 11 spacecraft it is postulated that an unbound massive particle has a cosmological redshift $z = (c/v_0)H_0 t$, where c is the speed of light in vacuum, v_0 is the initial velocity of the particle, H_0 is Hubble's constant and t is the duration of time that the particle has been unbound. The increase in wavelength of the particle corresponds to a decrease in its speed by $\Delta v = -cH_0 t$. Furthermore, it is hypothesized that the solar system has escaped the gravity of the Galaxy as evidenced by its orbital speed and radial distance and by the visible mass within the solar system radius. This means that spacecraft which become unbound to the solar system would also be galactically unbound and subject to the Hubble law. This hypothesis and the extended Hubble law may explain the anomalous acceleration found to be acting upon the unbound Pioneer 10 & 11 spacecraft. Thus, the Pioneer anomaly may be a counter example to the dark matter hypothesis.

Because photons have a speed which make them unbound to the Galaxy, it is predicted that the navigation beam in open space would undergo a cosmological redshift in its frequency which would be detectable with modern clocks.

Keywords Hubble law · Anomalous acceleration · Dark matter · MOND · Carmeli cosmology

1 Introduction

In this paper we will attempt to explain the anomalous acceleration found in the Pioneer 10 & 11 spacecraft as being due to the expansion of the universe. To accomplish this task, the Hubble law, which applies to the wavelength of light from distant galaxies, is extended to include unbound massive particles. It is well known that microscopic particles by way of their

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de Broglie wavelength display wave interference phenomena identical to light waves. We argue that the expanding universe, by the Hubble law, can have an effect on the de Broglie wavelength of a galactically unbound massive particle analogous to the way it increases the wavelength of a photon. The increase in wavelength of the particle corresponds to a decrease in its velocity.

In order for the Pioneer spacecraft to be unbound to the Galaxy it is necessary to hypothesize that the solar system itself is already galactically unbound. Evidence is given in support of this concept.

2 Newtonian System in an Expanding Universe

In a paper by Anderson [1] it was found that a Newtonian system of neutrally or electrically charged particles in an expanding Einstein–de Sitter universe can be described by (Ref. [1], Eq. (9))

$$
m_A \partial_{t_S}^2 \mathbf{x}_A + 2 \frac{\epsilon_H}{\epsilon_S} \partial_{t_S} \partial_{t_H} m_A \mathbf{x}
$$

$$
= -\frac{1}{R^3} \sum_{B \neq A} \frac{m_A m_B}{r_{AB}^3} \mathbf{r}_{AB} - 2 \frac{\epsilon_H}{\epsilon_S} m_A \frac{\partial_{t_H} R}{R} \partial_{t_H} \mathbf{x}_A + \mathcal{O}(\epsilon_S^2, \epsilon_H^2), \tag{1}
$$

where there are particles labeled $A, B, C, \ldots$, in the multiparticle system, m_A is the mass of the Ath particle whose coordinates are the vector $\mathbf{x}_A$, m_B is the mass of the Bth particle whose coordinates are the vector $\mathbf{x}_B$, etc., and $\mathbf{r}_{AB} = \mathbf{x}_A - \mathbf{x}_B$. We have also the scale factor $R(t_H) = (t_H/t_0)^{2/3}$ with t_H the parametrized cosmic time, t_0 the time at the present epoch where the Hubble time at the current epoch is $T_H = R(t)/\dot{R}(t) = (3/2)t_0$. The short-hand form of the partial derivative is expressed $\partial_t = \partial/\partial t$. Defining the time T_L as the typical time for light to travel across the system of particles, and the time T_S as a characteristic time (typical orbital period) of the system, then $\epsilon_S = T_L/T_S$ and $\epsilon_H = T_L/T_H$, and from these are defined the times $t_S = \epsilon_S t$ and $t_H = \epsilon_H t$ in terms of the cosmic time t.

Anderson made two conclusions from (1). First, if the system of particles is small such that $\epsilon_H \ll \epsilon_S$ then

$$
r^3 \omega = \frac{\alpha}{R^3} \quad \text{and} \quad r \omega = \frac{\beta}{R}, \tag{2}
$$

where α and β are constants, r is the comoving coordinate distance between the masses, and ω is the orbital angular velocity. From this it follows that

$$
\omega = \text{const} \quad \text{and} \quad Rr = \text{const}, \tag{3}
$$

which means that the orbital frequencies ω and radial distances Rr of the particles in the system are not changed by the expanding universe. This would be the situation for a small bound system of particles where Newton's laws apply, which is obtained from (1) with $\epsilon_H \ll \epsilon_S$,

$$
m_A \partial_{t_S}^2 \mathbf{x}_A = -\frac{1}{R^3} \sum_{B \neq A} \frac{m_A m_B}{r_{AB}^3} \mathbf{r}_{AB}. \tag{4}
$$

In the other case when $\epsilon_H \approx \epsilon_S$ there is an effect of the expansion upon the system of particles due to the two terms coupled by ϵ_H/ϵ_S in (1). For example, this condition is

satisfied by an unbound two particle system of a large mass (Sun) and a small mass (comet or spacecraft) which is in hyperbolic orbit, because in this case the system size is "infinite" so that the light travel time T_L across the system and the characteristic period T_S of the system are both comparable to the Hubble time T_H. Then the second term on the right hand side of (1) works out to

$$-2\frac{\epsilon_H}{\epsilon_S}m_A\frac{\partial_{t_H}R}{R}\partial_{t_H}m_A\mathbf{x}_A = -\frac{4}{3}\frac{\epsilon_H}{\epsilon_S}m_A\frac{\partial_{t_H}\mathbf{x}_A}{t_H}, \tag{5}$$

where

$$\frac{\partial_{t_H}R}{R} = \frac{\partial_{t_H}(t_H/t_0)^{2/3}}{(t_H/t_0)^{2/3}} = \frac{2}{3}\frac{1}{t_H}. \tag{6}$$

Substituting this result into (1), and assuming that $\partial_{t_S}\partial_{t_H}m_A\mathbf{x} = 0$, the equation of motion of a particle of mass m_A in unbound orbit (when $\epsilon_H \approx \epsilon_S$) is given by

$$m_A\partial_{t_S}^2\mathbf{x}_A = -\frac{1}{R^3}\sum_{B\neq A}\frac{m_A m_B}{r_{AB}^3}\mathbf{r}_{AB} - \frac{4}{3}\frac{\epsilon_H}{\epsilon_S}m_A\frac{\partial_{t_H}\mathbf{x}_A}{t_H}. \tag{7}$$

In the next section we look at what the Hubble law says about massive particles, and then make a connection with (7).

3 Hubble Law for a Massive Particle

The de Broglie [3] wavelength λ for a massive particle of momentum p is given by

$$\lambda = \frac{h}{p}, \tag{8}$$

where h is Planck's constant. This relation is also true for photons.

In the expanding universe we assume that the Hubble law for photons is also valid for galactically unbound massive particles. Then, for a particle which had an initial wavelength of λ_0 at (cosmological) redshift 0 but is now at redshift z, its wavelength λ is given by

$$\lambda = (1+z)\lambda_0. \tag{9}$$

By (8) this can be written in momentum form

$$\frac{h}{p} = (1+z)\frac{h}{p_0}, \tag{10}$$

$$p = \frac{p_0}{1+z}, \tag{11}$$

$$\Delta p = p - p_0 = \frac{-zp_0}{1+z}. \tag{12}$$

For $z \ll 1$ this implies

$$\Delta p \approx -zp_0. \tag{13}$$

The conclusion of the Pioneer 10 & 11 report (Ref. [2], Eq. (54)) was that there was an anomalous constant acceleration on the spacecraft directed toward the Sun of magnitude $(8.74 \pm 1.33) \times 10^{-8}$ cm s^{-2}. It was also noted (Ref. [2], Sect. C) that this value is approximately equal to the speed of light times the Hubble constant, cH_0. During these measurements the motion of either spacecraft was directed nearly radially outward from the Sun, hence the reported direction of the anomalous acceleration is also consistent with it being directed along the line of and in opposition to the motion of the spacecraft. With these experimental facts, we postulate that:

(P-1) *For $z \ll 1$ the change in momentum of an unbound particle due to the expansion of the universe is given by*

$$\Delta p \approx -zp_0 = -m_0 c H_0 t, \tag{14}$$

where m_0 is the mass of the particle at redshift 0 and t is the duration of time since the particle has been unbound.

This implies that for a massive particle the redshift

$$z = \frac{m_0 c}{p_0} H_0 t. \tag{15}$$

For a photon the momentum is $p_0 = m_0 c$, so (15) gives

$$z = H_0 t = \frac{H_0}{c} r, \tag{16}$$

which is recognized as the Hubble law for the redshift of light observed from a galaxy at distance $r = ct$, where t is the duration of time since the (unbound) photon left the galaxy.

For a massive particle moving at non-relativistic speed, $m \approx m_0$, so divide (14) by the mass to get the cosmological velocity shift

$$v_z(t) = \Delta v = -z v_0 = -c H_0 t, \tag{17}$$

from which we get the redshift relation for a massive particle in terms of its initial velocity

$$z = \frac{c}{v_0} H_0 t. \tag{18}$$

For larger redshift, though still assuming non-relativistic velocities, from (12) and (18) we *assume* that the cosmological velocity shift is given by

$$v_z(t) = \frac{-z v_0}{1+z} = \frac{-c H_0 t}{1 + (c/v_0) H_0 t}. \tag{19}$$

Assuming that the Hubble law for massive particles is related to (7) then from (17) we will assume the relation

$$\frac{v_z(t)}{t} = -c H_0 = -\frac{4}{3} \frac{\epsilon_H}{\epsilon_S} \frac{|\partial_{t_H} \mathbf{x}_A|}{t_H}. \tag{20}$$

Inherent in the assumption of (20) is that the rate of change of the position vector $\partial_{t_H} \mathbf{x}_A$ is decreasing, hence the negative sign multiplies the magnitude of the vector.

The final thing we need in our theory is a reason why a spacecraft which is unbound to the solar system would also be unbound to the Galaxy and therefore be in the category $\epsilon_H \approx \epsilon_S$ and therefore governed by (7).

4 Hypothesis that the Solar System Has Escaped the Gravity of the Galaxy

The solar system is at a distance [4] of $R_0 = (8 \pm 0.4)$ kpc from the center of the Galaxy. Its orbital circular speed is $V_0 = (220 \pm 15)$ km/s. The total visible mass M_{vis} of the Galaxy (Ref. [5], Table 4, Model 4) is composed of the Galaxy's bulge and disk masses, with $M_{bulge} \approx 0.364 \times 10^{10} M_\odot$ and $M_{disk} \approx 4.16 \times 10^{10} M_\odot$, giving

$$M_{vis} \approx (4.52 \pm 0.90) \times 10^{10} M_\odot, \tag{21}$$

where the error in (21) is this author's expression of an uncertainty of $\pm 20\%$. The total visible mass within the Sun's orbit would be less than this. If we assume that V_0 is greater than the escape velocity at R_0 then this implies that the Galaxy mass M_0 within the solar radius would be

$$M_0 < \frac{V_0^2 R_0}{2G} = (4.55 \pm 0.84) \times 10^{10} M_\odot. \tag{22}$$

The right hand side approximately equals the total visible mass in (21), suggesting that the solar system velocity V_0 is greater than the escape velocity for the entire visible Galaxy.

This discussion suggests that it is not unreasonable to hypothesize that:

(H-1) *The solar system Galactic orbital speed exceeds escape velocity at its current radius.*

This hypothesis avoids the need for any substantial halo of dark matter to bind the solar system with velocity V_0. This suggests that the Sun is in the Galaxy because, according to (19), the expansion of the universe over time continually decreases the solar system's velocity. This may help to account for why a galaxy does not fly apart even though the orbital speeds of objects in it exceed the Newtonian escape velocity of the visible mass. However, the theory of the dynamics of spiral galaxies is beyond the scope of this paper (cf. [6, 7]).

5 Pioneer 10 & 11 Anomalous Velocities and Accelerations

By the hypothesis (H-1) the solar system is unbound to the Galaxy. This implies that any spacecraft which escapes from the solar system will also have escaped from the Galaxy. The Pioneer 10 & 11 spacecraft [2] have escaped the solar system gravity and therefore by (H-1) are unbound to the Galaxy. Hence, their velocities will be cosmologically shifted as in (17), so that their anomalous velocity and acceleration are respectively given by

$$v_P(t) = v_z(t) = -cH_0t, \tag{23}$$

$$a_P = cH_0. \tag{24}$$

These would represent the anomalies found in the analysis of the spacecraft tracking data. However, with the value [2] of $a_P = (8.74 \pm 1.33) \times 10^{-8}$ cm/s^2 this would require that $H_0 = a_P/c = (90.0 \pm 13.7)$ km/s/Mpc, measured at a redshift $z = 0$. This is 6% to 44% larger than the currently accepted value of 72 km/s/Mpc. (It is not unlikely that if the spin histories (Ref. [2], Sect. D) of both spacecraft were properly accounted for [8], then the acceleration related to Hubble's constant would be closer to $a_P(0) = (7.84 \pm 0.01) \times 10^{-8}$ cm/s^2 which would correspond to $H_0 = (80.1 \pm 0.1)$ km/s/Mpc.) The relation (24) for a_P corresponds in principle to a minimum acceleration in the universe [9] having a finite value $a_{min} = c/\tau$, where the Hubble–Carmeli time constant $\tau \approx H_0^{-1}$.

6 Transmission of Light Beam Between a Body Bound to the Solar System and an Unbound Body

Let us review the Pioneer 10 & 11 anomaly with a little more detail. From our postulate (P-1) and the extended Hubble law (18), we analyse the frequency changes in a light (radio) signal sent between the solar system barycenter (SSB) and a spacecraft which is in unbound motion near to the solar system. Since we hypothesize (H-1) that the solar system has escaped the Galaxy, then the unbound spacecraft is also galactically unbound. Thus, the spacecraft motion relative to the solar system is described by (7), though more precisely by general relativity theory plus the extra term due to the expansion. The bodies in the solar system are bound and obey Newton's equation of motion (4), though more precisely general relativity theory.

The frequency shift upon the photons in the navigation beam due to gravity cancels out during a round trip. However, because of their speed c, these photons are unbound even to the Galaxy so that their frequencies will decrease according to (16). This cosmological redshift in the beam frequency in open space is actually independent of the H-1 hypothesis.

For simplicity of argument, we assume the unbound spacecraft is moving radially outward from the SSB at a velocity $v(t)$ based on the parametrized post-Newtonian approximation (PPN) and any other known effects upon objects in deep solar system orbit, such as due from solar radiation pressure and planetary dust. With respect to the observer fixed to the SSB the entire round trip of a light beam of initial frequency ν_0 originating at the SSB, traversing the distance to the spacecraft, reflecting off of the spacecraft and traveling back and received at the SSB, will have its frequency transformed on the outward journey by the factor $(1 - z_1)$ due to the effect of the expansion on the photon beam signal, where $z_1 = H_0 \Delta t_1$ is the redshift during the travel time Δt_1 to the spacecraft. At the spacecraft the beam is received and retransmitted, incurring the double factor $(1 - v(t)/c - v_z(t)/c)^2$ in change to its frequency, where from (17), $v_z(t) = -cH_0t$ is directed parallel to the spacecraft time dependent velocity $v(t)$. On the return trip the beam is again altered by the expansion factor of $(1 - z_2)$, where $z_2 = H_0 \Delta t_2$ is the redshift during the travel time Δt_2 back to the SSB. The roundtrip effect is

$$\nu_{\text{obs}}(t) = \nu_0(1 - z_1)(1 - v(t)/c - v_z(t)/c)^2(1 - z_2), \tag{25}$$

$$\approx \nu_0(1 - 2v(t)/c + 2H_0t - H_0\Delta t_1 - H_0\Delta t_2), \tag{26}$$

to first order in $v(t)/c$, $v_z(t)/c$, z_1 and z_2. If we limit the transmission times to and from the spacecraft to half day each of $\Delta t \approx 5 \times 10^4$ s, with $H_0 = 90$ km/s/Mpc $= 2.916 \times 10^{-18}$ s^{-1}, then $z_1 \approx z_2 \approx H_0 5 \times 10^4 \approx 1.46 \times 10^{-13}$. Whereas, for t a duration of a year or more, $v_z(t)/c = H_0t > 9.2 \times 10^{-11}$. Thus for this analysis we can ignore the effects of z_1 and z_2 on the beam signal since they are more than two orders of magnitude smaller than $v_z(t)/c$.

If we define the model expected observed frequency of the received light beam

$$\nu_{\text{model}}(t) = \nu_0(1 - 2v(t)/c), \tag{27}$$

then with this and (26) we have, in the Deep Space Network (DSN) negative format (Ref. [2], Eq. (15)), the anomalous frequency (blue) shift

$$\Delta\nu(t)_{\text{DSN}} = -(\nu_{\text{obs}}(t) - \nu_{\text{model}}(t)) = -2\nu_0 H_0 t, \tag{28}$$

which agrees with the Pioneer 10 & 11 result (Ref. [2], Eq. (15)) if we have that

$$H_0 = a_P/c. \tag{29}$$

7 Transmission of Light Beam Between Two Bound Solar System Bodies

For bodies bound in the solar system the separation between the bodies are determined by general relativity, there being no further separation of the bodies nor perturbation of the orbital periods due to the expansion of the universe. The PPN determined velocity $v(t)$ for this case is only the radial component along the line of sight from the SSB to the bound spacecraft, as this is the component which contributes in the Doppler effect upon the beam. As before, the photons in the navigation beam are not bound to the Galaxy and so are subject to the effect of the expansion.

Following the same line of reasoning as before, the round trip effect on the beam frequency is, from (25) with $v_z(t) = 0$,

$$\nu_{\text{obs}}(t) = \nu_0(1 - z_1)(1 - v(t)/c)^2(1 - z_2), \tag{30}$$

$$\approx \nu_0(1 - 2v(t)/c - H_0\Delta t_1 - H_0\Delta t_2), \tag{31}$$

to first order in $v(t)/c$, z_1 and z_2. Aside from the effects of z_1 and z_2, this is the usual result obtained between bounded bodies in the solar system. From (31), the anomalous frequency (red) shift is (in DSN format)

$$\Delta\nu(t)_{\text{DSN}} = -[\nu_{\text{obs}}(t) - \nu_0(1 - 2v(t)/c)] = \nu_0 H_0\Delta t, \tag{32}$$

where $\Delta t = \Delta t_1 + \Delta t_2$ is the round trip light travel time. The current accuracy of atomic clocks [10] is better than one part in 10^{15}. If a spacecraft is placed in a circular orbit of 28 AU about the Sun (within Neptune's orbit, where the solar radiation pressure acceleration will be $<5 \times 10^{-8}$ cm/s^2 for Pioneer 10 type spacecraft/antenna) the round trip travel time of a light signal sent to the spacecraft would be about $\Delta t \approx 2.8 \times 10^4$ s. From (32) this corresponds to an anomalous frequency redshift ratio of the returned signal of

$$\Delta\nu(t)_{\text{DSN}}/\nu_0 = H_0\Delta t \approx (2.916 \times 10^{-18}\,\text{s}^{-1})2.8 \times 10^4\,\text{s} \approx 8.2 \times 10^{-14}, \tag{33}$$

which is 82 times larger than the clock accuracy. This would be detectable on a statistical basis.

8 Conclusion

It seems natural to extend the Hubble law to the realm of massive particles by way of the associated de Broglie wave of the particle. Then unbound particles would exhibit shifts in their wavelengths due to the expansion of the universe. This shift would be detectable as a decrease in momentum (velocity) of the particle. It is a claim of this paper that the anomalies found in the Pioneer spacecraft navigation is a detection of this cosmological velocity shift. But, in order for the spacecraft to be unbound galactically it was necessary to hypothesize (H-1) that the solar system is not bound to the Galaxy. This hypothesis is based on the fact that there appears to be an insufficient amount of visible mass in the Galaxy to bind the solar system. Thus the Pioneer spacecraft anomaly may be a counter example to the hypothesis that large amounts of dark matter exists in the Galaxy. A further effect, which is independent of the H-1 hypothesis, would be a cosmological redshift in the frequency of the communication signal in open space.

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